5 For each of the following functions $f$, sketch on the same set of axes the graphs of $y = f(x - 2) + 3$, $y = f(x + 4) - 4$, and $y = f(x)$.

- $f(x) = x^2$ (Red graph)
- $f(x) = e^x$ (Green graph)
- $f(x) = \frac{1}{x}$ (Blue graph)

6 Copy these functions and then draw the graph of $y = f(x - 2) - 3$.

- $y = f(x)$
- $y = f(x)$
7. Given that the following points lie on \( y = f(x) \), find the coordinates of the point each moves to under the transformation \( y = 3f(2x) \):

\[ f(x) = 3 \cdot \frac{2x}{3} \cdot \frac{3}{2} \]

- \( f(3) = -5 \)
- \( g(\frac{3}{2}) = 3f\left(2\left(\frac{3}{2}\right)\right) = 3f(3) = -15 \)
- \( g\left(\frac{3}{2}\right) = 3f\left(\frac{3}{2}\right) = -15 \)
- \( g\left(\frac{3}{2}\right) = 3f\left(\frac{3}{2}\right) = -15 \)

- \( f(1) = 2 \)
- \( g(x) = 3f(2x) \)
- \( g\left(\frac{1}{2}\right) = 3f\left(2\left(\frac{1}{2}\right)\right) = 3f(1) = \frac{3}{6} = \frac{1}{2} \)

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- \( g\left(\frac{1}{2}\right) = 3f\left(\frac{1}{2}\right) = \frac{3}{6} = \frac{1}{2} \)
7  a  Given that the following points lie on $y = f(x)$, find the coordinates of the point each moves to under the transformation $y = 3f(2x)$:
   i  (3, -5)  ii  (1, 2)  iii  (-2, 1)

b  Find the points on $y = f(x)$ which are moved to the following points under the transformation $y = 3f(2x)$:
   i  (2, 1)  ii  (-3, 2)  iii  (-7, 3)

$$g(x) = 3f(2x)$$
$$g(-1) = 3f(2(-1)) = 3f(1) = 3$$

$$f(-2) = 1$$
$$(-1, 3)$$

7  a  Given that the following points lie on $y = f(x)$, find the coordinates of the point each moves to under the transformation $y = 3f(2x)$:
   i  (3, -5)  ii  (1, 2)  iii  (-2, 1)

b  Find the points on $y = f(x)$ which are moved to the following points under the transformation $y = 3f(2x)$:
   i  (2, 1)  ii  (-3, 2)  iii  (-7, 3)

$$g(2) = 1$$
$$g(x) = 3f(2x)$$
$$g(2) = 3f(2 \cdot 2) = 1$$
$$3f(4) = 1$$
$$f(4) = \frac{1}{3}$$

$$(-6, \frac{2}{3})$$
$$(4, \frac{1}{3})$$
7  a Given that the following points lie on \( y = f(x) \), find the coordinates of the point each moves to under the transformation \( y = 3f(2x) \):
   
   \[ \begin{align*}
   &i \quad (3, -5) \\
   &ii \quad (1, 2) \\
   &iii \quad (-2, 1)
   \end{align*} \]

   b Find the points on \( y = f(x) \) which are moved to the following points under the transformation \( y = 3f(2x) \):
   
   \[ \begin{align*}
   &i \quad (2, 1) \\
   &ii \quad (-3, 2) \\
   &iii \quad (-7, 3)
   \end{align*} \]

\[
\begin{align*}
  g(-7) &= 3 \\
  g(x) &= 3f(2x) \\
  g(-7) &= 3f(2(-7)) = 3 \\
  3f(-14) &= 3 \\
  f(-14) &= 1
\end{align*}
\]

6  The function \( y = f(x) \) is transformed to \( h(x) = f(-x) \).

   a Find the image points on \( h(x) \) for the following points on \( f(x) \):
   
   \[ \begin{align*}
   &i \quad (2, -1) \\
   &ii \quad (0, 3) \\
   &iii \quad (-1, 2)
   \end{align*} \]

   b Find the points on \( f(x) \) corresponding to the following points on \( h(x) \):
   
   \[ \begin{align*}
   &i \quad (5, -4) \\
   &ii \quad (0, 3) \\
   &iii \quad (2, 3)
   \end{align*} \]
6 The function \( y = f(x) \) is transformed to \( h(x) = f(-x) \).

a Find the image points on \( h(x) \) for the following points on \( f(x) \):
   i \((2, -1)\) \((-2, -1)\) ii \((0, 3)\) \((0, 3)\) iii \((-1, 2)\) \((-1, 2)\)

b Find the points on \( f(x) \) corresponding to the following points on \( h(x) \):
   i \((5, -4)\) \((-5, -4)\) ii \((0, 3)\) \((0, 3)\) iii \((2, 3)\) \((-2, 3)\)
The graph of \( f(x) = 3x^3 - 2x^2 + x + 2 \) is translated to its image \( g(x) \) by the vector \( \left( \frac{1}{-2} \right) \). Write the equation of \( g(x) \) in the form \( g(x) = ax^3 + bx^2 + cx + d \).

\[
\begin{align*}
g(x) &= 3(x-1)^3 - 2(x-1)^2 + (x-1) + 2 - 2 \\
    &= 3\left(x^3 - 3x^2 + 3x - 1\right) - 2\left(x^2 - 2x + 1\right) + (x-1) \\
    &= 3x^3 - 9x^2 + 9x - 3 - 2x^2 + 4x - 2 + x - 1 \\
    &= 3x^3 - 11x^2 + 14x - 6
\end{align*}
\]
\[
\begin{array}{c}
\binom{1}{1} \\
\binom{1}{2} \\
\binom{1}{3} \\
\binom{1}{4} \\
\binom{1}{5} \\
\binom{1}{6} \\
\end{array}
\]

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
5 & 6 & 10 & 5 \\
6 & 15 & 20 & 15 & 4
\end{array}
\]

\[
(a+b)(a^2+2ab+b^2) \\
\frac{a^3+2a^2b+ab^2}{a^2b+2ab^2+b^3} \\
\frac{a^3+3a^2b+3ab^2+b^3}{a^3+3a^2b+3ab^2+b^3}
\]
3. The graph of \( f(x) = 3x^3 - 2x^2 + x + 2 \) is translated to its image \( g(x) \) by the vector \( \left( \frac{1}{2} \right) \). Write the equation of \( g(x) \) in the form \( g(x) = ax^3 + bx^2 + cx + d \).

\[
g(x) = f(x-1) - 2
g(x) = 3(x-1)^3 - 2(x-1)^2 + (x-1) + 2 - 2 \quad \text{one right}
= 3(x^3 - 3x^2 + 3x - 1) - 2(x^2 - 2x + 1) + (x - 1)
= 3x^3 - 9x^2 + 9x - 3 - 2x^2 + 4x - 2 + x - 1
= 3x^3 - 11x^2 + 14x - 6
\]

\[
(a+b)^2 = a^2 + 2ab + b^2
\]

\[
(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3
\]

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
1 & 2 & 3 & 4 \\
1 & 3 & 6 & 4 \\
1 & 4 & 10 & 10 \\
1 & 5 & 10 & 10 \\
\end{array}
\]
Any questions from the transformation chapter?
If \( f(x) = 5 - x - x^2 \), find in simplest form:

- \( a \) \( f(-1) \)
- \( b \) \( f(x-1) \)
- \( c \) \( f\left(\frac{x}{2}\right) \)
- \( d \) \( 2f(x) - f(-x) \)

\[
2\left(5-x-x^2\right) - \left(5-(-x)-(-x)^2\right)
\]

\[
10-2x-2x^2 - \left(5+x-x^2\right)
\]

\[
10-2x-2x^2 - 5-x+x^2
\]

\[-x^2 - 3x + 5\]